

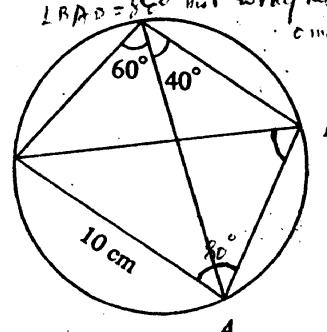
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89 CE Maths I-1

Solution	Marks	Remarks
<p>1. (a) Increase percentage = <math>(\frac{1000}{8000} \times 100)\%</math>  <math>= 12.5\%</math></p> <p>(b) His savings = <math>\\$9000 \times \frac{3}{10}</math>  <math>= \\$2700</math></p>	<p>1A  <math>\frac{1A}{2}</math></p> <p>1A  <math>\frac{1A}{2}</math></p>	<p>for <math>\frac{1000}{8000}</math>  Accept 12.5</p>
<p>2. (a) <math>x + 1 &gt; \frac{1}{5}(3x + 2)</math>  <math>5x - 3x &gt; 2 - 5 \dots \dots \dots</math>  <math>2x &gt; -3</math>  <math>x &gt; -\frac{3}{2}</math></p> <p>(b) Furthermore, if <math>-4 \leq x \leq 4</math>, then the range of <math>x</math> is  <math>-\frac{3}{2} &lt; x \leq 4</math>.</p>	<p>1M  <math>\frac{1A}{2}</math></p> <p>2A  <math>\frac{2}{2}</math></p>	<p>OR  <math>x - \frac{3}{5}x &gt; \frac{2}{5} - 1</math> 1M  <math>\frac{2}{5}x &gt; -\frac{3}{5}</math>  <math>x &gt; -\frac{3}{2}</math> 1A</p> <p>-1 if '=' incorrect  Accept graphical representation</p>
<p>3. (a) Since <math>(x + 1)</math> is a factor of <math>x^4 + x^3 - 8x + k</math>,  <math>(-1)^4 + (-1)^3 - 8(-1) + k = 0</math> <math>\checkmark</math> correct  <math>k = -8</math></p> <p>(b) <math>x^4 + x^3 - 8x - 8 = (x + 1)(x^3 - 8)</math>  <math>= (x + 1)(x - 2)(x^2 + 2x + 4)</math></p>	<p>1M  <math>\frac{1A}{2}</math></p> <p>1M+1A  <math>\frac{1A+1A}{2}</math></p> <p>4</p>	<p>1M for <math>(x+1) \times</math> cubic exp.  1A for <math>x^3 - 8 = (x-2)(x^2+2x+4)</math></p>
<p>OR <math>(2)^4 + (2)^3 - 8(2) - 8 = 0</math>  <math>\rightarrow x - 2</math> is another factor  <math>\therefore x^4 + x^3 - 8x - 8 = (x + 1)(x - 2)(x^2 + 2x + 4)</math> <math>\checkmark</math> correct  <math>\uparrow</math>  <math>\uparrow</math></p>	<p>1A+2A  <math>\frac{1M+2A}{2}</math></p>	<p>1M for <math>(x+1)(x-2) \times</math> quadratic exp.</p>

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89 CE Maths I-2

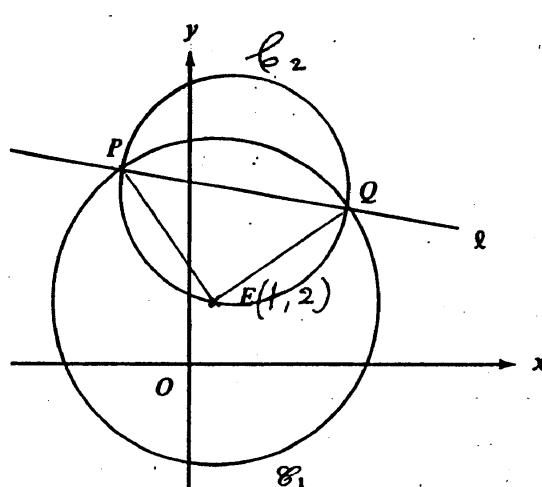


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89 CE Maths I-3

Solution	Marks	Remarks
<p>7. <math>3\tan\theta = 2\cos\theta</math></p> $3 \frac{\sin\theta}{\cos\theta} = 2\cos\theta$ $3\sin\theta = 2\cos^2\theta$ $3\sin\theta = 2(1 - \sin^2\theta)$ $\therefore 2\sin^2\theta + 3\sin\theta - 2 = 0 \dots \dots \dots \dots \dots \dots$ $(2\sin\theta - 1)(\sin\theta + 2) = 0$ $\sin\theta = \frac{1}{2} \text{ or } -2 \text{ (rejected)}$ <p>The solutions are <math>\theta = 30^\circ</math> or <math>150^\circ</math> (<math>\frac{\pi}{6}</math> or <math>\frac{5\pi}{6}</math>) [as <math>\cos 30^\circ</math> and <math>\cos 150^\circ \neq 0</math>].</p>	1M 1M 1A 1A 1A+1A <hr/> 7	Accept ' $\sin\theta = \frac{1}{2}$ ' or ' $\sin\theta = -2$ ' Deduct 1 for each extraneous solution.

Solution	Marks	Remarks
8. (a) $E = (1, 2)$	1A	$E = 1, 2$ $\boxed{P+1}$
(b) From $x + 7y - 40 = 0$ , we have $x = 40 - 7y$ (or $y = \frac{40 - x}{7}$ )	1	
Putting in $\ell_1$ , $(40-7y)^2 + y^2 - 2(40-7y) - 4y - 20 = 0$ $50y^2 - 550y + 1500 = 0$ $y^2 - 11y + 30 = 0$ (or $x^2 - 3x - 10 = 0$ ) $(y - 5)(y - 6) = 0$ $y = 5$ or $6$ (or $x = 5$ or $-2$ ) $x = 5$ or $-2$	1M 1A 1A 1A 1A 1A	$y = 5$ and $y = 6$ $\boxed{P+1}$
$\therefore P = (-2, 6), Q = (5, 5)$	1A	Accept $P = (5, 5)$ $Q = (-2, 6)$
	4	
(c) $\ell_2$ is given by $\frac{y - 6}{x + 2} \cdot \frac{y - 5}{x - 5} = -1$ i.e. $x^2 + y^2 - 3x - 11y + 20 = 0$	1M+1A 1A	OR Ctr. of $\ell_2 = (\frac{3}{2}, \frac{11}{2})$ radius = $\frac{5\sqrt{2}}{2}$ ( $= 3.54$ ) $\boxed{1A}$
	3	Eqt. of $\ell_2$ : $(x - \frac{3}{2})^2 + (y - \frac{11}{2})^2 = \frac{50}{4}$ $\boxed{1M+1A}$
(d) Putting $(x, y) = (1, 2)$ in L.H.S. of $\ell_2$ $1^2 + 2^2 - 3(1) - 11(2) + 20 = 0$	1M 1A	OR Slope of PE $\times$ slope of $QE = -1$
$\therefore \ell_2$ passes through E		
(As PQ is a diameter of $\ell_2$ ), $\angle PEQ = 90^\circ$ (Since PE = QE (radii of $\ell_1$ ),) $\angle EPQ = \frac{90^\circ}{2} = 45^\circ$	1M 1A 1A	OR Let $P = (-2, 6), Q = (5, 5)$ Slope of PQ = $-\frac{1}{7}$ Slope of PE = $-\frac{4}{3}$ $\tan \angle EPQ = \frac{-\frac{1}{7} - \frac{-4}{3}}{1 + \frac{1}{7} \times \frac{4}{3}} = 1$ $\angle EPQ = 45^\circ$
	4	OR $171.87^\circ - 126.87^\circ = 45^\circ$



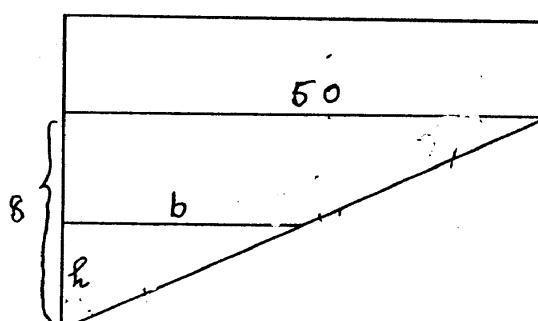
Solution	Marks	Remarks
9. (a) $\frac{k}{1} = \frac{\frac{1}{2}}{k}$ $k^2 = \frac{1}{2}$ $k = \frac{1}{\sqrt{2}}$ ( or $\frac{\sqrt{2}}{2}$ ) (as $k > 0$ )	1M  <hr/> 1A <hr/> <hr/> <hr/>	
(b) $T(n) = \left(\frac{1}{\sqrt{2}}\right)^{n-1}$ [ or $\frac{1}{(\sqrt{2})^{n-1}}$ , $2^{-\frac{n-1}{2}}$ , etc.]	1M+1A <hr/> <hr/> <hr/>	$\frac{1}{\sqrt{2}}^{n-1}$ p.p.
(c) Sum to infinity = $\frac{1}{1 - \frac{1}{\sqrt{2}}}$  $= \frac{\sqrt{2}}{\sqrt{2} - 1}$  $= \frac{\sqrt{2}(\sqrt{2} + 1)}{(\sqrt{2} - 1)(\sqrt{2} + 1)}$  $= 2 + \sqrt{2}$ .....	1M+1A  <hr/> 1M <hr/> 1A <hr/> <hr/>	
(d) No. of terms in the product = $\frac{2n - 1 - 1}{2} + 1 = n$  $T(1) \times T(3) \times T(5) \times \dots \times T(2n-1)$  $= 1 \times \frac{1}{2} \times \frac{1}{4} \dots \times \left(\frac{1}{\sqrt{2}}\right)^{2n-2}$ [ or $1 \times \frac{1}{(\sqrt{2})^2} \times \frac{1}{(\sqrt{2})^4} \times \dots \times \frac{1}{(\sqrt{2})^{2n-2}}$ ]  $= 1 \times \frac{1}{2} \times \frac{1}{2^2} \times \dots \times \frac{1}{2^{n-1}}$ $= \frac{1}{2^{1+2+\dots+(n-1)}} \dots$  $= \frac{1}{2^{\frac{-n(n-1)}{2}}} \quad [ \text{or } 2^{\frac{-n(n-1)}{2}}, \text{ etc.} ]$	1A  <hr/> 1M  <hr/> 1M+1A <hr/> <hr/>	1M for summing index as A.P.

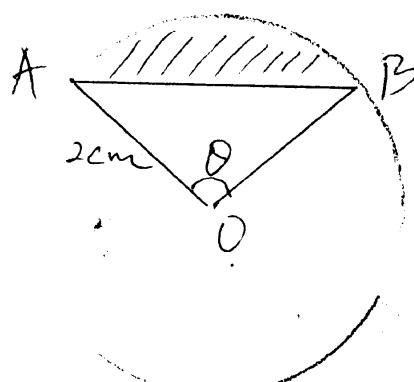
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89 CE Maths I-6

Solution	Marks	Remarks
10. (a) $AB' = 10\cos 45^\circ$ $= 5\sqrt{2}\text{m}$ (or $\frac{10}{\sqrt{2}}$ , 7.07107) $AC' = 10\cos 30^\circ$ $= 5\sqrt{3}\text{m}$ (8.66025)	1A    <hr style="width: 100px; margin-left: 0; border: 1px solid black;"/> 1A <hr style="width: 100px; margin-left: 0; border: 1px solid black;"/>	Any figure roundable to 7.07
(b) $BC = \sqrt{10^2 + 10^2}$ $= 10\sqrt{2}\text{m}$ (14.14214) $BB' = 10\sin 45^\circ$ $= 5\sqrt{2}\text{m}$ (7.07107) $CC' = 10\sin 30^\circ$ $= 5\text{m}$	1A    <hr style="width: 100px; margin-left: 0; border: 1px solid black;"/> 1A <hr style="width: 100px; margin-left: 0; border: 1px solid black;"/>	No unit - 1m for u - 1
	1A <hr style="width: 100px; margin-left: 0; border: 1px solid black;"/>	
(c) Let D be the foot of the perpendicular from C to BB'. $BD = (5\sqrt{2} - 5)\text{m}$ (= 2.07107) $B'C' = CD$ $= \sqrt{(10\sqrt{2})^2 - (5\sqrt{2} - 5)^2}$ $= \sqrt{125 + 50\sqrt{2}}\text{m}$ (= 13.9897)	1M    <hr style="width: 100px; margin-left: 0; border: 1px solid black;"/> 1M <hr style="width: 100px; margin-left: 0; border: 1px solid black;"/> 1A <hr style="width: 100px; margin-left: 0; border: 1px solid black;"/> 3 <hr style="width: 100px; margin-left: 0; border: 1px solid black;"/>	Accept figures roundable to 13.9 - 14.0
(d) By the cosine rule, $\cos B'AC' = \frac{50 + 75 - (125 + 50\sqrt{2})}{2 \times 5\sqrt{2} \times 5\sqrt{3}}$ (= $-\frac{1}{\sqrt{3}}$ , -0.57735) 1M $\angle B'AC' = 125^\circ$ (125.264)	1A <hr style="width: 100px; margin-left: 0; border: 1px solid black;"/>	$124^\circ - 125^\circ$
Area of the shadow = $\frac{1}{2} \times 5\sqrt{2} \times 5\sqrt{3} \sin 125.264^\circ$ $= 25\text{m}^2$	1M <hr style="width: 100px; margin-left: 0; border: 1px solid black;"/> 1A <hr style="width: 100px; margin-left: 0; border: 1px solid black;"/> 4	For $\Delta = \frac{1}{2} ab \sin C$ 25.0 - 25.4

Solution	Marks	Remarks
11. (a) Area of cross-section = $\frac{50}{2} (2 + 10) = 300\text{m}^2$ Vol. of water = $20 \times 300 = 6000\text{m}^3$	2CA 1M+1A	1M for Vol. = Area of cross-section x width OR $\frac{20 \times 50 \times 2}{2} + \frac{1}{2} (50 \times 8) \times 20$
(b) (i) When the depth of water at the deeper end is 8m, the cross-section of water is a triangle of area $\frac{8 \times 50}{2} = 200\text{m}^2$ . Vol. of water left = $200 \times 20 = 4000\text{m}^3$ .	2A	Drop in water level = 2m Water pumped out = $2 \times 50 \times 20 = 2000\text{m}^3$ 1A Water left = $4000\text{m}^3$ 1A
(ii) Vol. of water pumped out in 8 hours $= (0.125)^2 \pi \times 3600 \times 8 \times 3$ $= 1350\pi \text{ m}^3$ $= 4241\text{m}^3$ (correct to the nearest $\text{m}^3$ ) (4241.15)	1M+1A 1A	1M for area of cross-section
(iii). Vol. of water left after 8 hrs = $6000 - 4241$ $= 1759\text{m}^3$  When the depths of water are 8m and h m, the corresponding cross-sections of water are two similar triangles with bases 50m and b m. $\frac{b}{h} = \frac{50}{8}$ or $b = \frac{50}{8} h$ $\therefore \frac{1}{2} b \times h \times 20 = 1759$ $\frac{20}{2} \times \frac{50}{8} h^2 = 1759$ $h = 5.305 = 5.3$ (correct to 1 d.p.)	1M 1A 1M 1A 1A 1M 1M 1A 10	$\left(\frac{h}{8}\right)^2 = \frac{1759}{4000}$



Solution	Marks	Remarks																											
12. (a) (i) Area of $\triangle OAB = \frac{1}{2}(2)(2)\sin\theta = 2\sin\theta \text{ cm}^2$	1A																												
(ii) The area is greatest when $\theta = \frac{\pi}{2} \approx 1.57$	1A	90° not acceptable																											
	2																												
(b) Area of sector $OAB = \frac{1}{2}(2)^2\theta = 2\theta \text{ (cm}^2)$ $2\theta = 2\sin\theta = 2$ $\therefore \theta - \sin\theta - 1 = 0$	1A 1M 1A 3																												
(c) $f(0) = 0 - 0 - 1 < 0$ $f(3) = 3 - \sin 3 - 1 (= 1.859) > 0$ $\therefore 0 < \alpha < 3$ <i>If wrong, 1A is not given.</i> <i>If omitted, no 1A</i>	1M	For sub. $f(0)$ , $f(3)$ Accept graphical method																											
	1A 2																												
(d)																													
<table border="1"> <thead> <tr> <th>Interval</th> <th>Mid-value <math>\theta</math></th> <th><math>f(\theta)</math></th> </tr> </thead> <tbody> <tr> <td><math>0 &lt; \alpha &lt; 3</math></td> <td>1.5</td> <td>-</td> </tr> <tr> <td><math>1.5 &lt; \alpha &lt; 3</math></td> <td>2.25</td> <td>+</td> </tr> <tr> <td><math>1.5 &lt; \alpha &lt; 2.25</math></td> <td>1.875 (1.88)</td> <td>-</td> </tr> <tr> <td><math>1.875 &lt; \alpha &lt; 2.25</math></td> <td>2.063 (2.06)</td> <td>+</td> </tr> <tr> <td><math>1.875 &lt; \alpha &lt; 2.063</math></td> <td>1.969 (1.97)</td> <td>+</td> </tr> <tr> <td><math>1.875 &lt; \alpha &lt; 1.969</math></td> <td>1.922 (1.92)</td> <td>-</td> </tr> <tr> <td><math>1.922 &lt; \alpha &lt; 1.946</math></td> <td>1.946 (1.95)</td> <td>+</td> </tr> <tr> <td colspan="2"><math>1.922 &lt; \alpha &lt; 1.946</math></td><td></td> </tr> </tbody> </table>	Interval	Mid-value $\theta$	$f(\theta)$	$0 < \alpha < 3$	1.5	-	$1.5 < \alpha < 3$	2.25	+	$1.5 < \alpha < 2.25$	1.875 (1.88)	-	$1.875 < \alpha < 2.25$	2.063 (2.06)	+	$1.875 < \alpha < 2.063$	1.969 (1.97)	+	$1.875 < \alpha < 1.969$	1.922 (1.92)	-	$1.922 < \alpha < 1.946$	1.946 (1.95)	+	$1.922 < \alpha < 1.946$			1M+1A	1M Testing of sign at mid-value of suitable interval 1A Correct sign Correct choice of sub-interval
Interval	Mid-value $\theta$	$f(\theta)$																											
$0 < \alpha < 3$	1.5	-																											
$1.5 < \alpha < 3$	2.25	+																											
$1.5 < \alpha < 2.25$	1.875 (1.88)	-																											
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$1.875 < \alpha < 2.063$	1.969 (1.97)	+																											
$1.875 < \alpha < 1.969$	1.922 (1.92)	-																											
$1.922 < \alpha < 1.946$	1.946 (1.95)	+																											
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	1M																												
	1A																												
We see that $\alpha$ lies between 1.922 and 1.946. $\therefore \alpha = 1.9$ (correct to 1 d.p.)	1A 5																												
																													

Solution	Marks	Remarks
13. (a) Since $p + q = 1$ , putting $p = 3q$ $4q = 1$ $q = \frac{1}{4}$	1A <hr style="width: 100px; margin-left: 0;"/> 1A <hr style="width: 100px; margin-left: 0;"/>	optional <i>only</i> $q = \frac{1}{4}$ 1A.
(b) (i) The probability that the first ball drawn is black is $\frac{n}{10}$ . After a black ball has been drawn, the probability of drawing a second black ball is $\frac{n-1}{9}$ . $\therefore$ the probability that both balls are black	1A <hr style="width: 100px; margin-left: 0;"/> 1A <hr style="width: 100px; margin-left: 0;"/>	$\frac{n}{10} \times \frac{n-1}{9}$ 1A + 1M $\frac{n}{10} \times \frac{n-1}{9}$ wrong 1A + 1M
(ii) $\frac{n(n-1)}{90} > \frac{1}{3}$ ..... $3n^2 - 3n - 90 > 0$ $n^2 - n - 30 > 0$ $(n - 6)(n + 5) > 0$ $\therefore n > 6$ or $n < -5$	1M <hr style="width: 100px; margin-left: 0;"/> 1A <hr style="width: 100px; margin-left: 0;"/> 1A <hr style="width: 100px; margin-left: 0;"/>	Accept $n > 6$ with <i>error</i> <i>by letting n = 7, 8, 9, 10</i> 3A <i>all correct</i>
(c) The probability that the first ball drawn is red and the second is also red = $\frac{1}{2} \times \frac{4}{6}$ ( = $\frac{1}{3}$ ). The probability that the first is green and the second is red = $\frac{1}{2} \times \frac{3}{6}$ ( = $\frac{1}{4}$ ). $\therefore$ the probability that the ball drawn from N is red = $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$ .	1A <hr style="width: 100px; margin-left: 0;"/> 1A <hr style="width: 100px; margin-left: 0;"/> 1A <hr style="width: 100px; margin-left: 0;"/>	<i>no explanation</i> 1A <i>only expression</i> 1A <hr style="width: 100px; margin-left: 0;"/>

Solution	Marks	Remarks
14. (a).		1A for each line + 1A + 1A
	1A Region 4	±1 horizontal/vertical unit at (100, 0), (0, 100); (20, 0), (60, 80); (0, 20), (100, 20)
(b) (i) $z = 100 - x - y$	1A	
(ii) Cost of mixture = $6x + 5y + 4z$ = $6x + 5y + 4(100 - x - y)$ = $2x + y + 400$ dollars	1A 1A	
(iii) $400x + 600y + 400z \geq 44000$ $800x + 200y + 400z \geq 48000$ Putting $z = 100 - x - y$ , $y \geq 20$	1A 1A	
$2x - y \geq 40$		
Further, (as $z \geq 0$ , $100 - x - y \geq 0$ ) $x + y \leq 100$	1A	or least cost
(iv) Drawing the line $2x + y = 0$ in the figure, the least cost is attained when $x = 30$ , $y = 20$ . $\therefore x = 30$ , $y = 20$ , $z = 50$	1M 1A	Any line. Costs at (30, 20), (80, 20), $(\frac{140}{3}, \frac{160}{3})$ are 480, 580 and 546.7 (Any point)
	8	